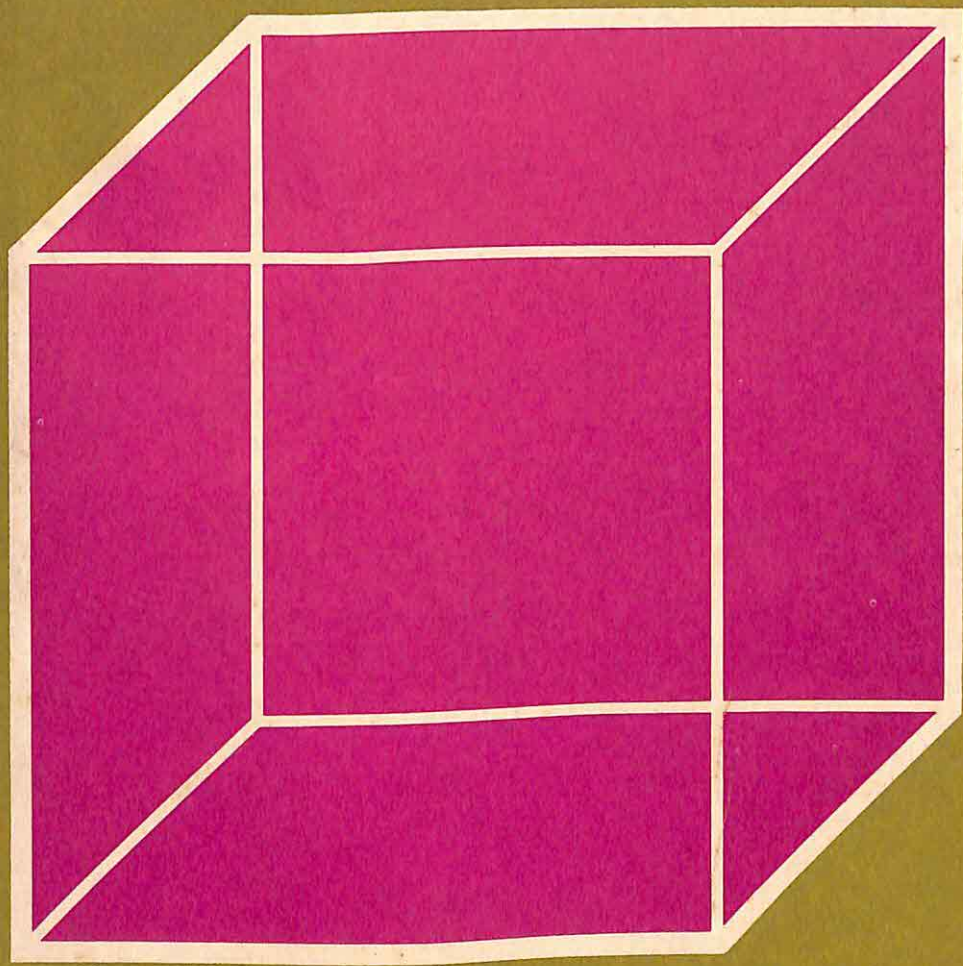


Mathematics in the Making 3

Looking at Solids

Stuart E Bell

218v



Longmans

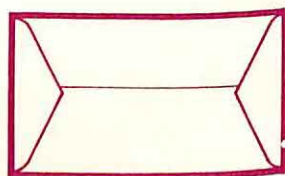
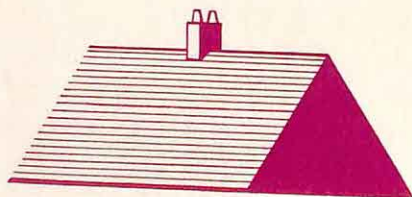
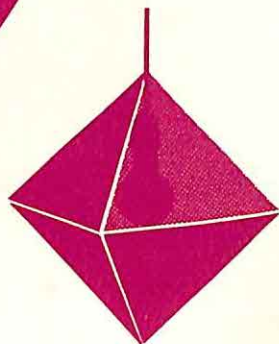
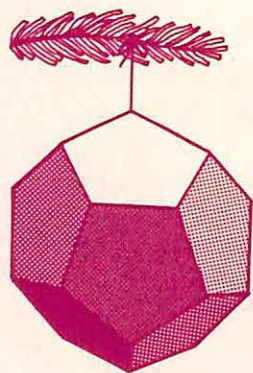
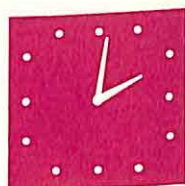
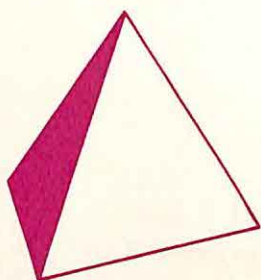
✓
2180
6-2-80

Stuart E Bell
Headmaster
Whipton Barton Junior School,
Exeter

in collaboration with
the School Mathematics Project
at Exeter School



Looking at Solids



Do you recognise all these things?

Can you say what shape each one is?

The flat ones are a set of POLYGONS.

Make a list of the polygons which you can recognise on the opposite page and write down the number of sides that each one has, like this.

Name of polygon	Number of sides
Triangle	3

Now look at the other objects.

How are they different from the set of polygons?

Are they flat?

Those which are not flat are known as **SOLIDS**.

Which solids do you recognise?



In this booklet we are going to discover how to construct some interesting solids.

Look at the sides of the set of solids. These are known as **FACES**.

Each of these faces, except for those on the cylinder, is a polygon.

Which polygons do you recognise?

Write down their names.

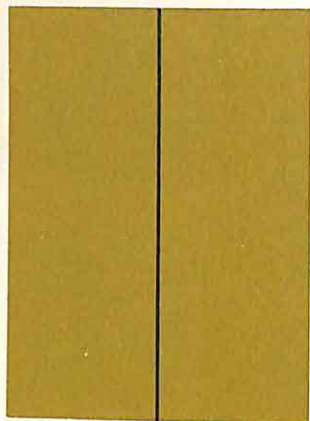
We call the corners of a solid **VERTICES** (pronounced *vertisees*).

When you have made some of these solids, you will be asked to count the faces, vertices and edges of each one.

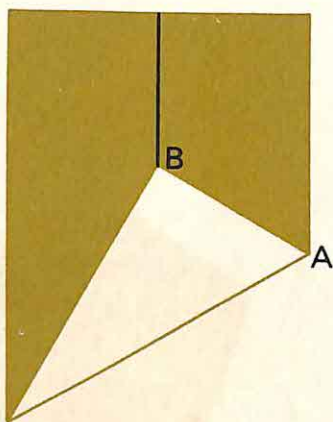
Let us now look more closely at some of the polygons that make the faces of the set of solids. One of the most common polygons is the triangle with equal sides and angles, known as the equilateral triangle.

Rotation and Angles, another booklet in this series, shows how to construct an equilateral triangle with a pair of compasses. Here is another method using paper folding.

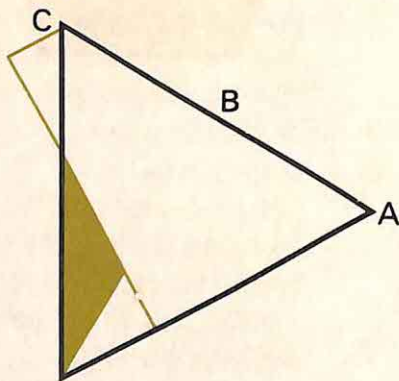
Stage 1. Fold the paper in half. (The paper must be rectangular.)



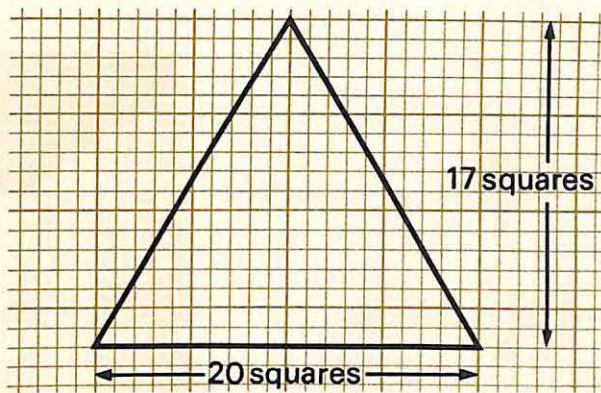
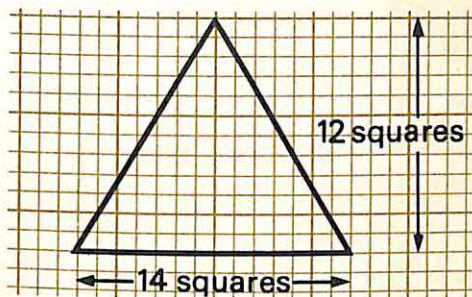
Stage 2. Fold the bottom right hand corner, B, on to the centre crease, so that the fold passes through the bottom left hand corner.



Stage 3. Fold the top of the piece of paper over so that the edge marked A-B is continued to point C. The equilateral triangle is outlined in black.

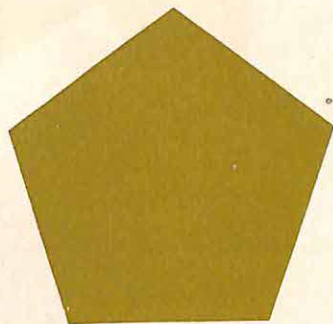


An approximate but quick way of drawing a large number of equilateral triangles is to use graph paper and to count the number of squares, like this.



The smaller triangle is suitable for making a solid which has a large number of faces.

The Pentagon



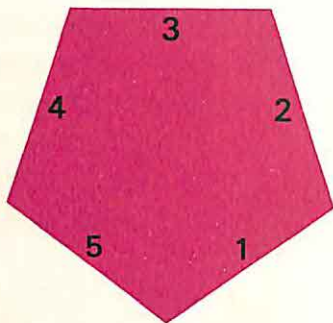
A five-sided polygon is called a **PENTAGON**.

Did you discover this shape when you examined the faces of the solids?

Trace the pentagon on to a piece of paper and cut it out. Fit it back into the hole you have made. Will it fit back in any other way? In how many ways will it fit? Will it fit if you turn it over?

6

Mark each edge of the pentagon like this.



Turn the pentagon round so that each edge is facing you in turn. Does it always look the same?

Now turn the pentagon round so that each of the vertices in turn is pointing towards you.

Does it always look the same?

Now examine the edges and angles of the pentagon.

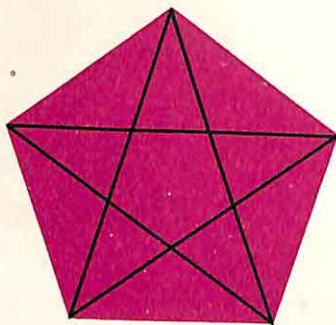
Are the edges the same length?

Are the angles the same size?

Draw lines to join the alternate vertices of the pentagon.

These lines are called diagonals. How many can you draw?

Are the diagonals the same length?

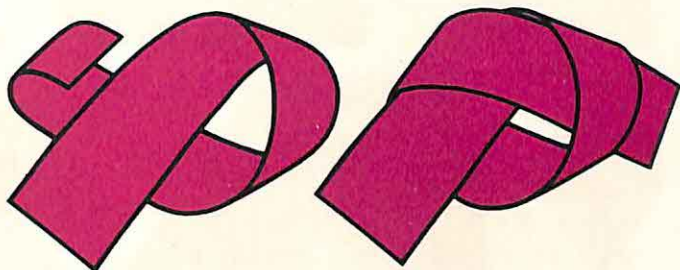


A polygon which has equal edges and angles, and which always looks the same when we turn it round, is called a **REGULAR POLYGON**.

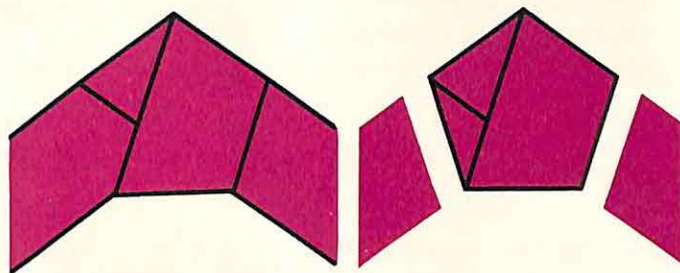
Is the pentagon we have traced a regular pentagon?

In *Rotation and Angles*, we discovered a method of making a regular pentagon by using a protractor. Another quick method is to tie a knot in a long strip of paper, like this.

Stage 1 The knot.



Stage 2 Flatten the knot, pull through the ends and cut them off.



Are all pentagons regular?
Can you draw one which is not?

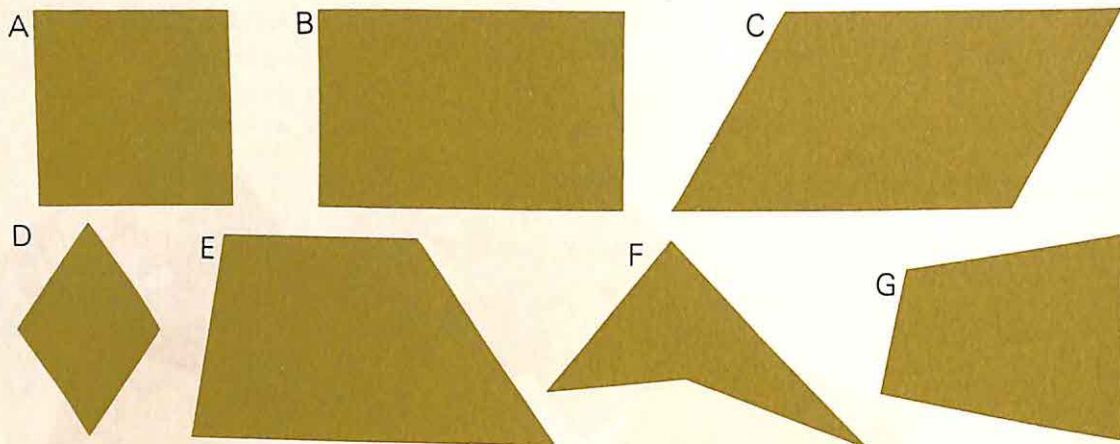
The Set of Quadrilaterals

Did you discover the square face on one of the solids you examined?

The square belongs to the set of four-sided polygons which are known as **QUADRILATERALS**. The square is a regular quadrilateral. Why? Squares can be easily drawn on graph or squared paper.

Can you make one by constructing right angles?

Here are several quadrilaterals.



1 Write down the names of the ones you recognise.

2 Which one is a regular polygon?

3 A and B are common shapes, but what about C? In what way is it like B and in what way is it different?

Shape C is known as a **PARALLELOGRAM** because its opposite sides are parallel.

4 What do you notice about the angles in shape C?

5 Shape B is the most common kind of parallelogram. Can you say why?

Now look at shape D. You probably know it as a diamond, but its usual name is a **RHOMBUS**.

A rhombus is a quadrilateral with equal sides.

What else do you notice are equal?

6 A square is a special kind of rhombus. Can you say why?

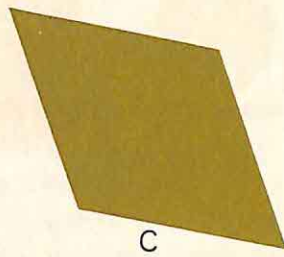
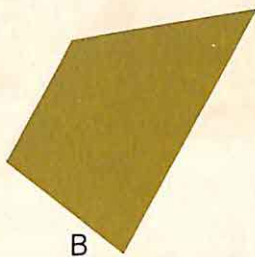
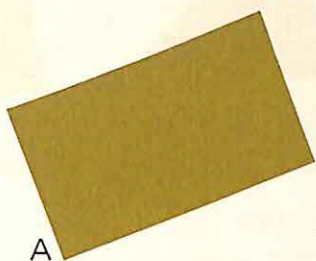
Now look at shape E. This quadrilateral is known as a **TRAPEZIUM**. A trapezium has one pair of parallel sides.

7 Draw a trapezium. Do not draw the one on page 8.

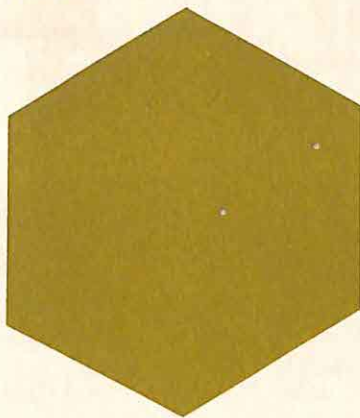
Shapes F and G have no special properties. Their sides and angles are all different and there are no parallel sides.

Here are some quadrilaterals seen from a different angle.

Name them.



This is a **HEXAGON**.



1 Is it regular? Give reasons for your answer.

2 How many vertices has it? You can construct a regular hexagon by fitting together a number of equilateral triangles. How many do you need?

3 How many sides has a 3d piece?

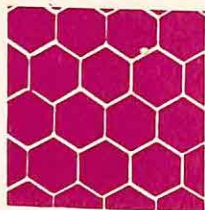
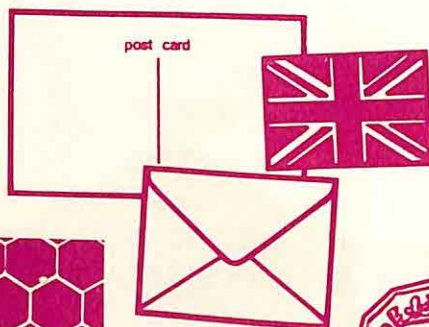
4 Is it regular? Give reasons for your answer.

A twelve sided polygon is called a **DODECAGON**.

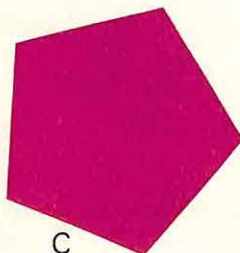
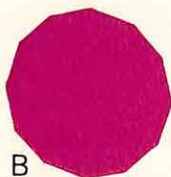
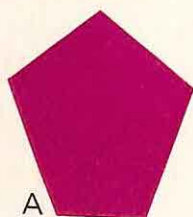
Make a table to show the number of vertices in the polygons you have studied. Write down some examples of these shapes being used in the world around you.

Name of shape	Number of vertices	Examples
Triangle		A roof
Trapezium		
Hexagon		
Rhombus		
Pentagon		
Parallelogram		
Dodecagon		
Rectangle		

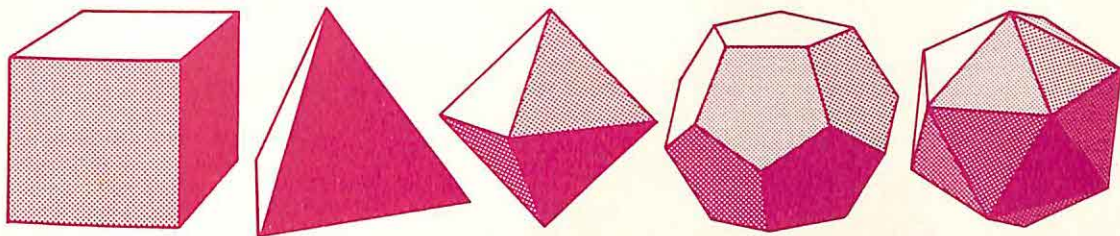
Make a collection of objects or pictures to show these shapes and arrange them in families.



Which of these polygons are regular?



Let us look more closely at some solids.



Can you see why?

Each of these solids is regular.

Regular solids have always interested mathematicians. They were first discovered before the time of Jesus, by the ancient Greeks, who found that there were only five solids which were regular. Because there were only five the Greeks believed that they had some special importance in the world in explaining some of the things which they did not then understand. They decided that the earth was made of cubes, and that water and air were made of two of the other regular solids. Nowadays we know that our world is not quite so simple as the Greeks believed. But they were not entirely wrong. Many of the minerals and rocks which are found in the earth are formed from crystals shaped like these solids.

If we were to examine salt crystals under a magnifying glass we should find that each one was shaped like a cube. Diamond crystals are shaped like the fourth solid.

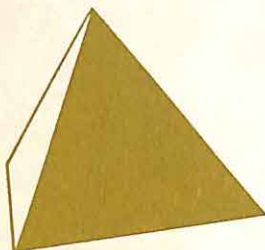
The regular solids are very beautiful and make lovely decorations when painted.

In the following pages you are going to learn how to make them.

You will need : thin white cardboard, cartridge paper, or graph paper, a ruler and pencil, a pair of compasses, scissors, and a quick-drying glue.

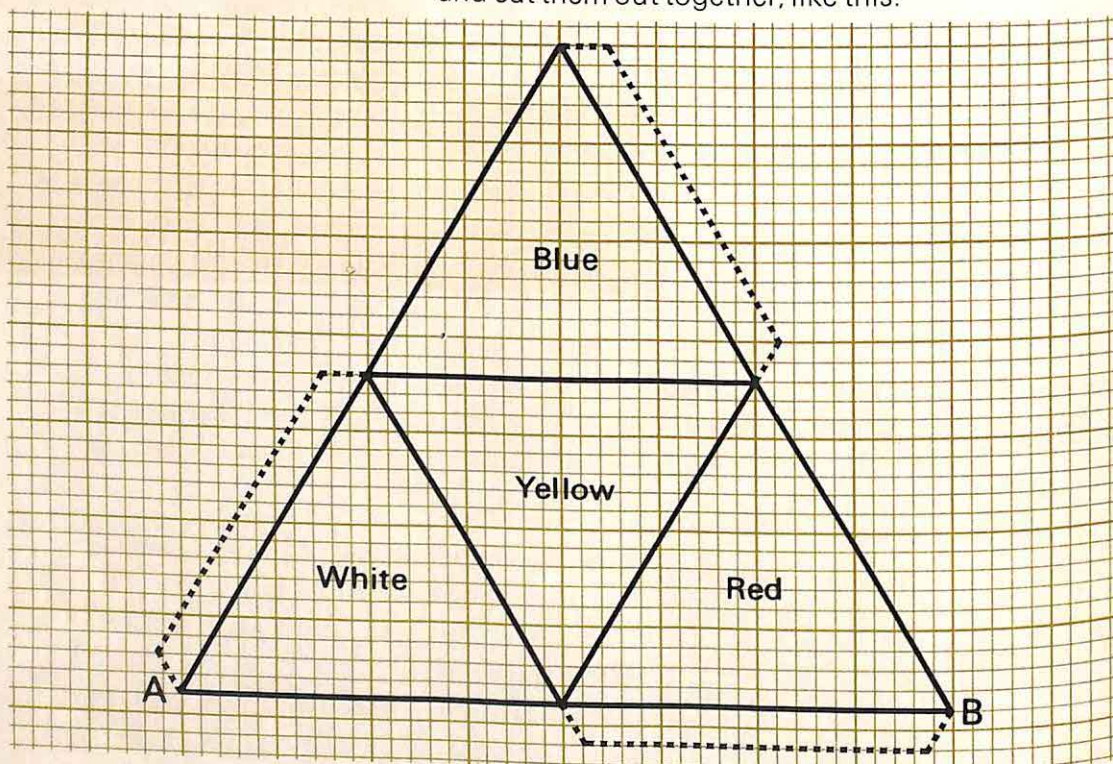
The Tetrahedron

A pyramid with four faces is called a
TETRAHEDRON.



What is the shape of
each face?

You could make this tetrahedron by joining four
equilateral triangles together, but an easier method
is to draw the triangles on the same piece of paper
and cut them out together, like this.



We call this the 'net' of the tetrahedron.

Draw the net of the tetrahedron. Construct the triangles by one of these methods : either—

(i) draw them on graph paper by counting squares ;
or

(ii) trace the large triangle on the right on to a thick piece of card. Cut it out.

Draw round it to make each of the triangles in the net.

Draw the line A—B first.

Write the colour names on each triangle.

Now you must score all the lines you have drawn on the net.

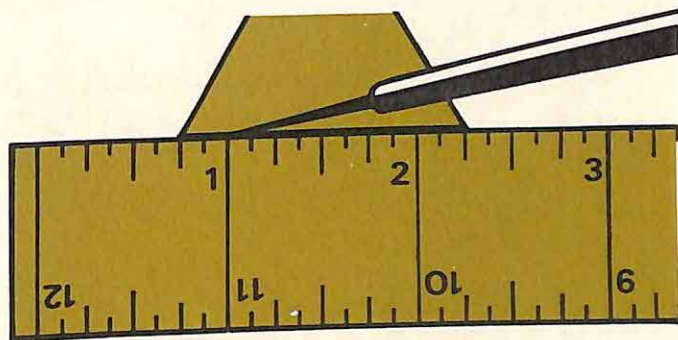
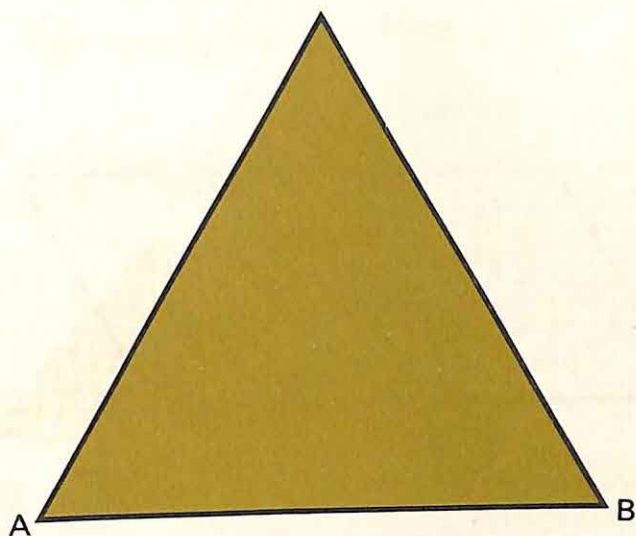
To do this, place the ruler along each line and draw the point of the compass along the line as if it were a pencil. This will make your model easier to fold.

You can fasten your tetrahedron together either by using sticky tape or by making tabs (shown on the net by dotted lines), which must be scored.

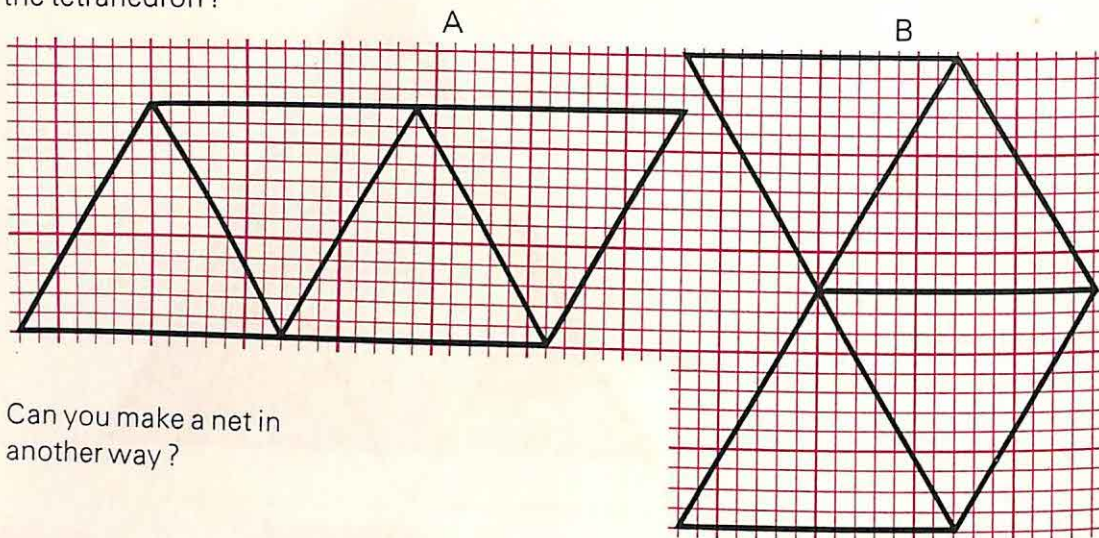
Colour your tetrahedron.

Now answer these questions

- 1 How many faces has the tetrahedron ?
- 2 How many vertices ?
- 3 How many edges ?
- 4 Is it regular ? How can you tell ?



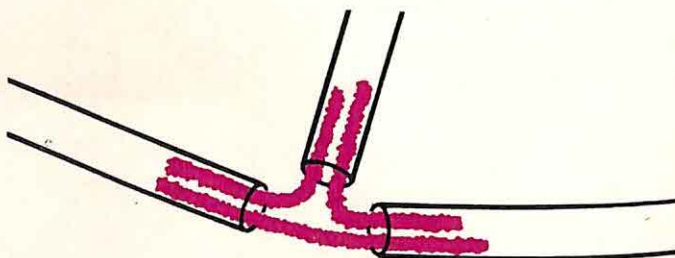
Which of these is a net of the tetrahedron?



Can you make a net in another way?

The skeleton of a tetrahedron and that of other solids can be made from pipe cleaners and straws.

Cut the pipe cleaners in half and use them to join the straws.



The Plaited Tetrahedron

A tetrahedron can also be made by plaiting. In this method, which is similar to that used for plaiting hair, each piece of the net is plaited alternately under and over the next piece.

This is a net for a plaited tetrahedron :

Draw and cut out this net, either by using graph paper or the cardboard triangle you cut out. You do not need tabs.

Number the triangles 1, 2, 3 and 4.

Score the black lines.

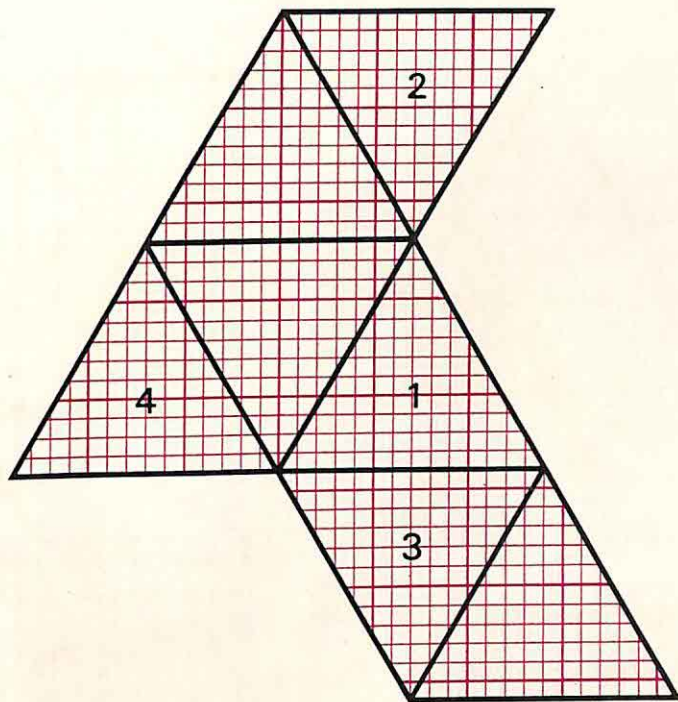
Step 1

Plait 1 over 2.

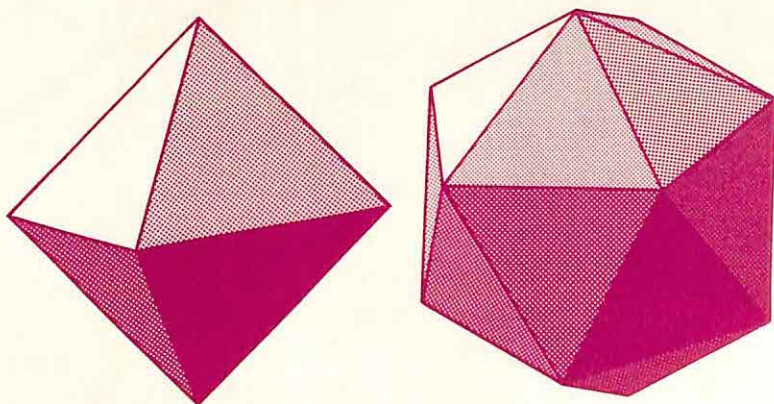
Step 2

Plait 3 over 4.

The last triangle is tucked in.



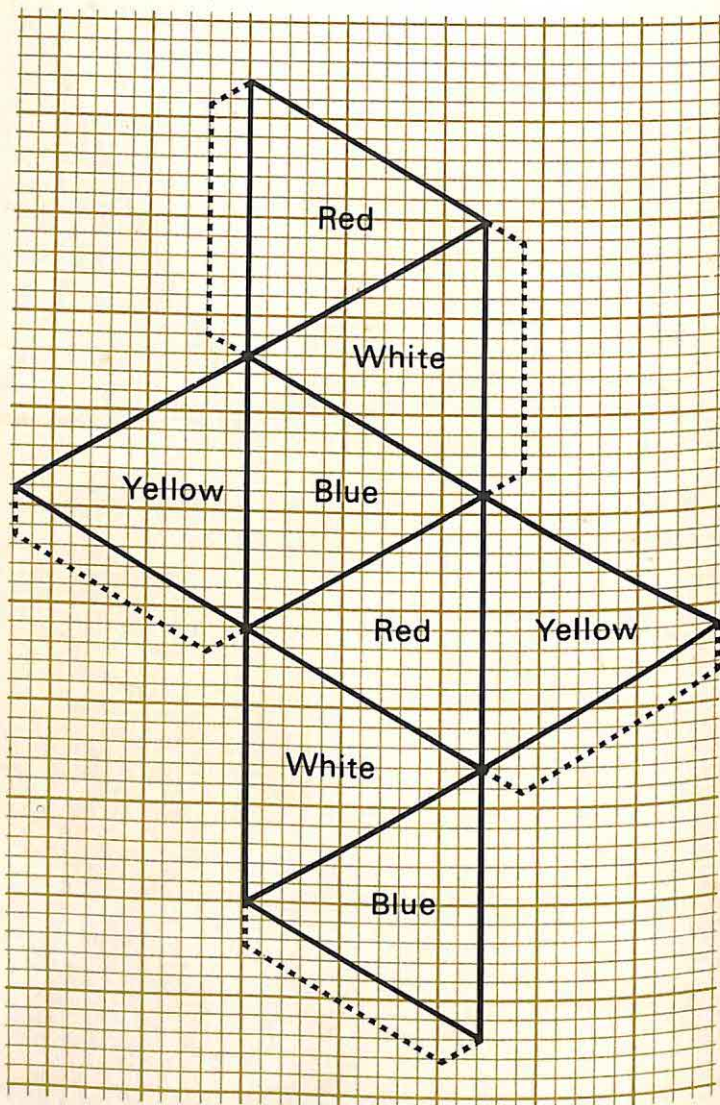
Here are two other solids constructed from equilateral triangles.



The Octahedron

Draw and cut out the net either by using graph paper or by drawing round your cardboard triangle. Paint the sides as shown. Score all the lines. Find out how to assemble the octahedron.

Here is the net of the **OCTAHEDRON**.

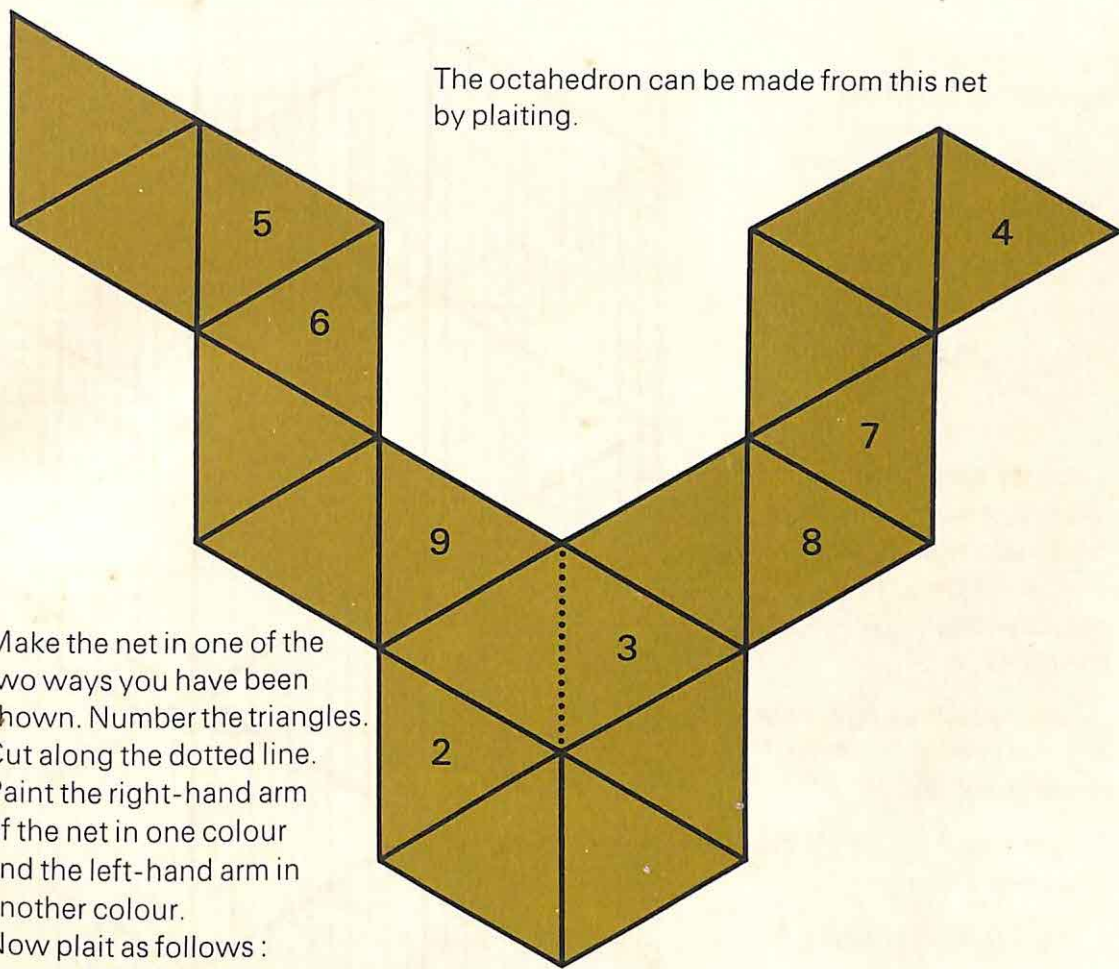


Now answer these questions :

- 1 How many faces has the octahedron ?
- 2 Which polygon's name begins with 'oct—' ? How many sides has it ?
- 3 How many vertices has the octahedron ?
- 4 How many edges ?
- 5 Does it look the same from every direction ? Is it regular ?
- 6 Can you make the net in a different way ?

The Plaited Octahedron

The octahedron can be made from this net by plaiting.



Make the net in one of the two ways you have been shown. Number the triangles. Cut along the dotted line. Paint the right-hand arm of the net in one colour and the left-hand arm in another colour.

Now plait as follows :

Step 1 Plait 3 over 2.

You have now made four faces of the octahedron.

Step 2 Plait 9 over 8.

Step 3 Plait 7 over 6.

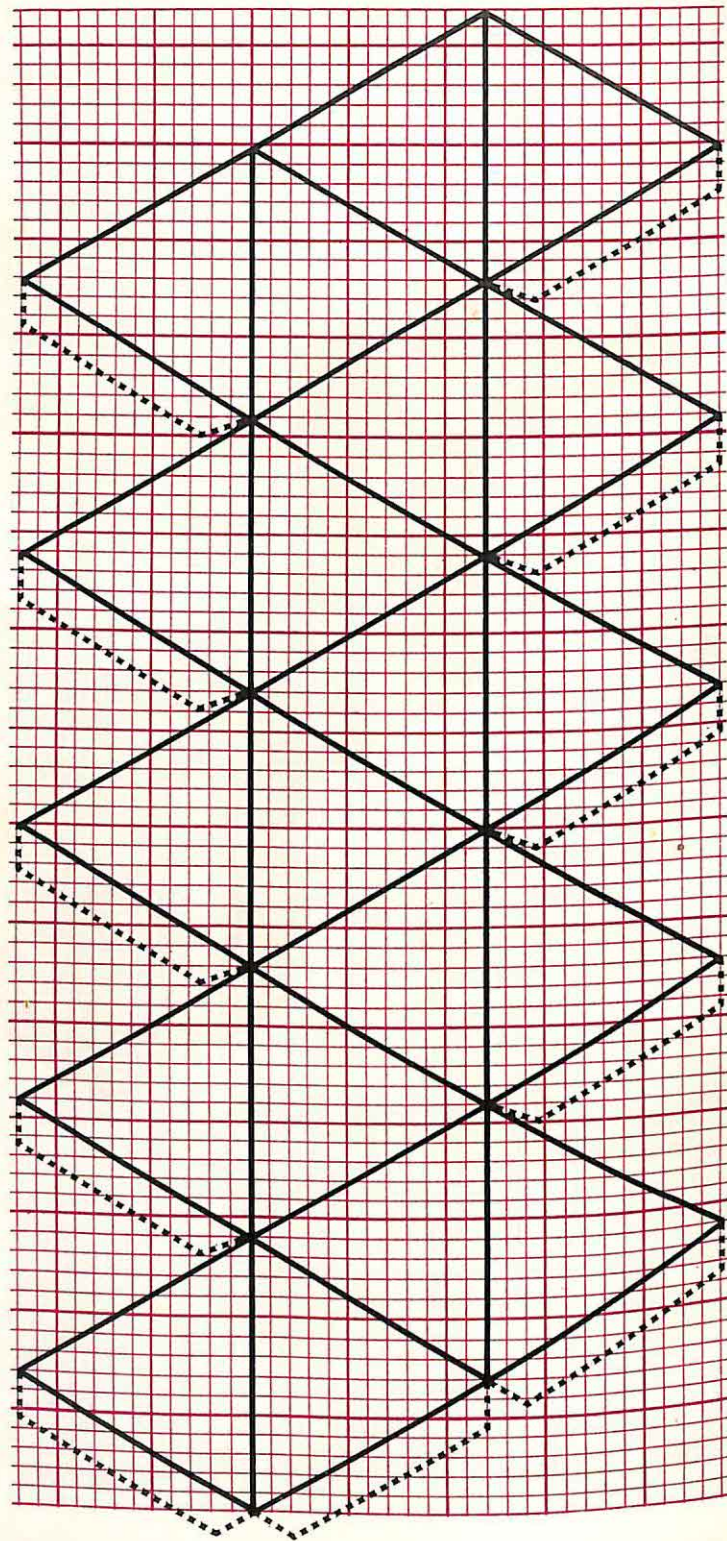
Step 4 Plait 5 over 4.

Complete the solid.

Make the skeleton of an octahedron from straws and pipe cleaners.

The Icosahedron

Here is the net of the ICOSAHEDRON.



Copy it using one of the two ways you have been shown. Score all the lines. Discover how to fit it together. Start off by joining the top faces.

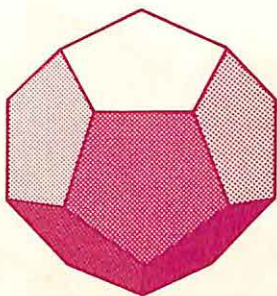
When you have completed the icosahedron, answer these questions.

- 1 How many faces has the icosahedron?
- 2 How many vertices?
- 3 How many edges?
- 4 Roll the icosahedron over several times. Does it always look the same? Give a reason for your answer.

Make the skeleton of an icosahedron with straws and pipe cleaners.

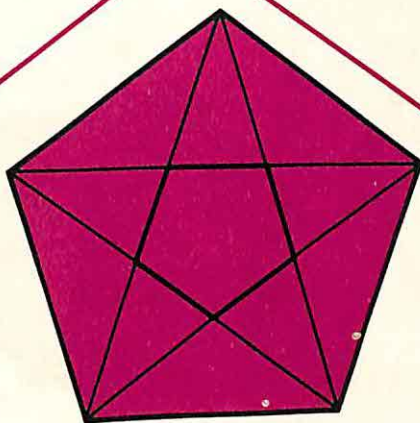
The Dodecahedron

What shape is each face of this solid, the
DODECAHEDRON ?

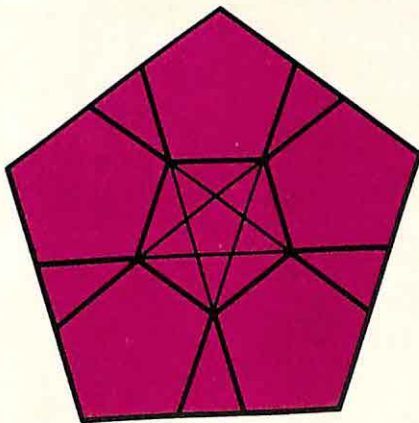


The net of the dodecahedron is made up of regular pentagons.

Draw a large regular pentagon, either by tying a knot as you have been shown, or by using the method shown in *Rotation and Angles*, or by tracing the large pentagon on this page. Join alternate corners of the pentagon like this. What shape have you formed in the centre?

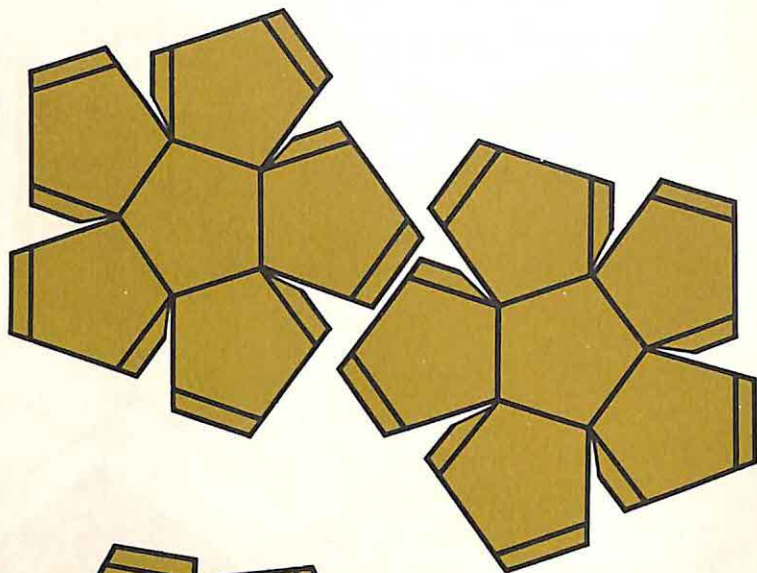


Now join alternate corners of the small pentagon and continue the lines until they meet the outer pentagon, like this.



You have now made six small pentagons which form half the net of the dodecahedron.

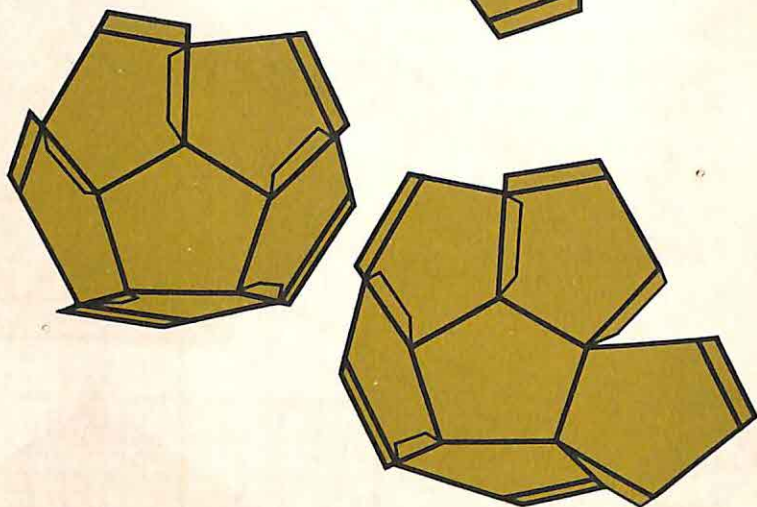
Make the other half in exactly the same way. Here are the two half nets showing the position of the tabs, which are different on the second half.



Score all the lines and colour or mark each face.

Assemble each half to form a cup shape and fit the two cups together to make the dodecahedron.

So that you can get your fingers inside, leave one of the pentagons on one cup unfastened until last of all.



Answer these questions about the dodecahedron.

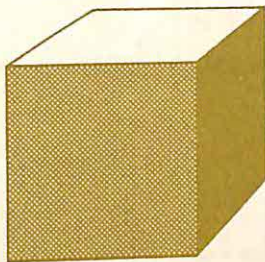
- 1 How many faces has the dodecahedron?
- 2 How many vertices?
- 3 How many edges?
- 4 Is the shape regular?
How do you know?

Make the skeleton of the dodecahedron with pipe cleaners and straws.

The Cube

You will probably know this regular solid best of all.

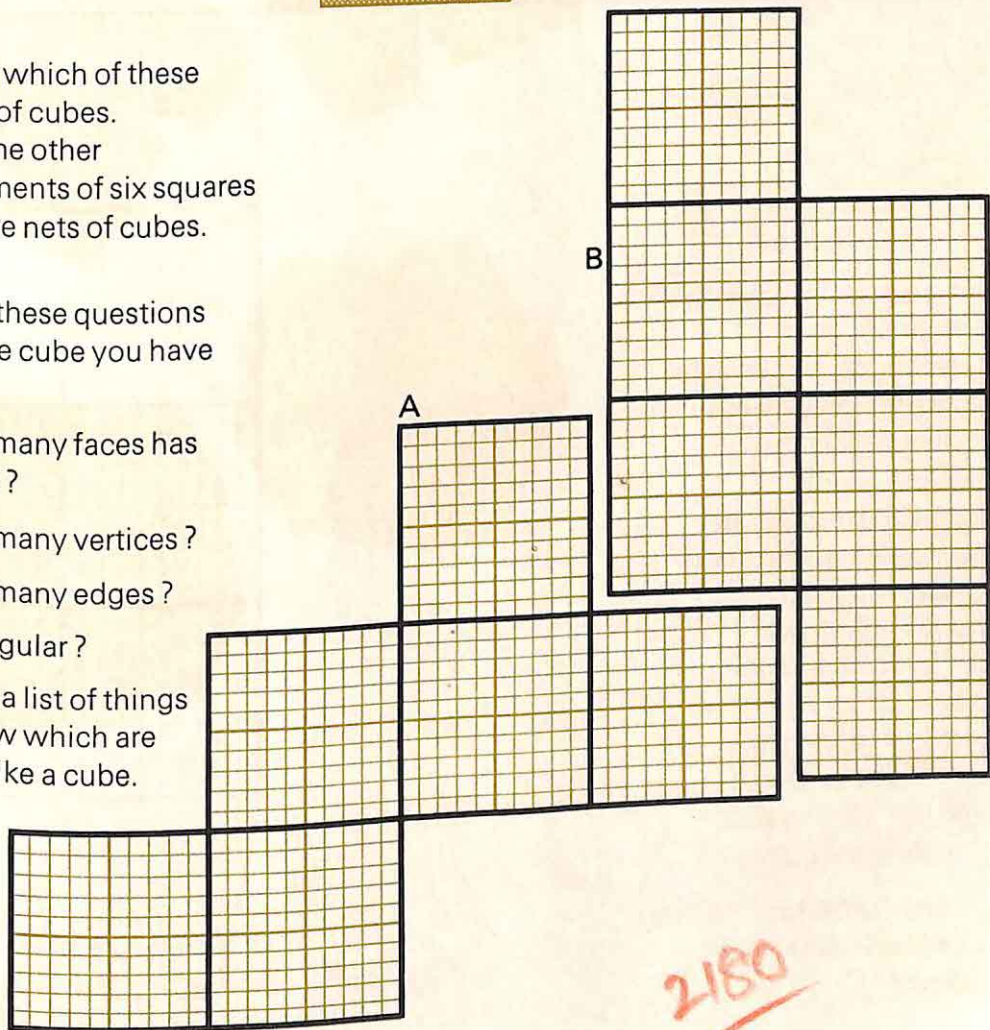
What is the shape of each face? How many faces are there?



Find out which of these are nets of cubes.
Find some other arrangements of six squares which are nets of cubes.

Answer these questions about the cube you have made:

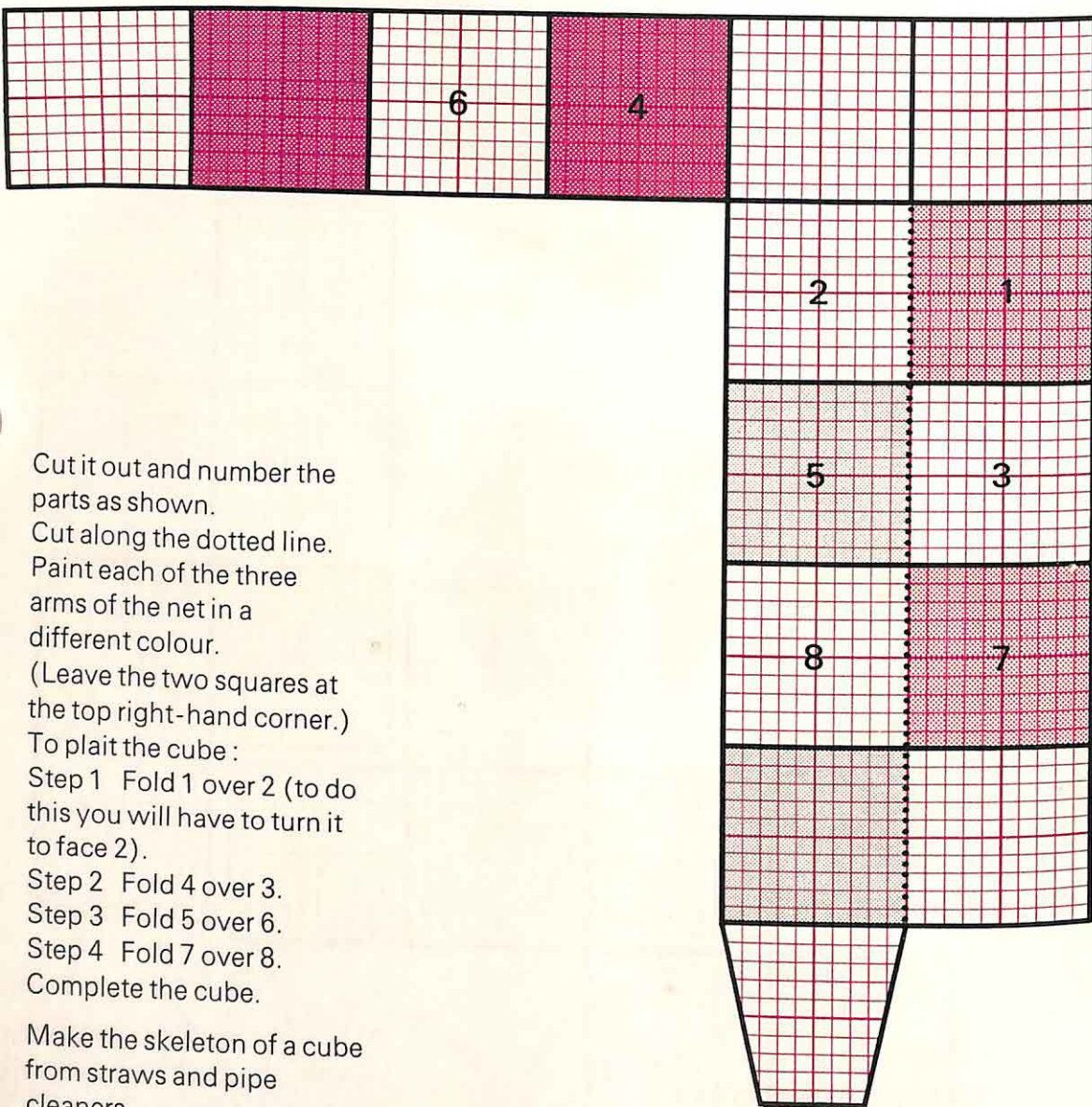
- 1 How many faces has the cube?
- 2 How many vertices?
- 3 How many edges?
- 4 Is it regular?
- 5 Make a list of things you know which are shaped like a cube.



2180

The Plaited Cube

Here is the net of the plaited cube.



Make a table to show all the things you have discovered about the five regular solids, like this.

Name	Number of faces	Number of vertices	Number of edges
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

Add the number of faces to the number of vertices in the tetrahedron.

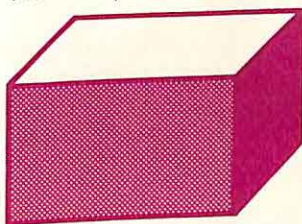
Compare your answer with the number of edges.

1 Which is bigger and by how much?

Now do the same thing with the other solids.

2 What do you find?

Here is another solid shape, a cardboard box. This shape is very similar to the cube.



It is called a CUBOID.

Can you see why?

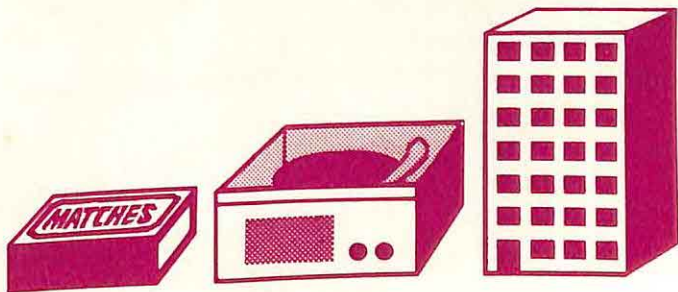
3 Is it regular? Give a reason for your answer.

Add up the number of faces and vertices.

Now find the number of edges.

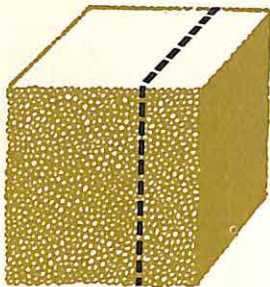
Do you get a result similar to that for question 1?

4 Make a list of things you know which are shaped like a cuboid and draw them.



Prisms

Let us suppose we had a cube of sugar, or a bath cube, and we were to cut off one end, like this.



What would be the shape of the cut end ?

If we were to make a number of cuts parallel to the first, would the shape of the cut end always be the same ?

If you are not sure about the answer, you can find out by making a cube out of Plasticine and cutting it for yourself.

This is an unsharpened pencil.



What is the shape of the end ?

If we were to cut it exactly in half, what would be the shape of the cut end ? If we were to make a series of cuts parallel to the first, would the shape of the cut end always be the same ?

Perhaps you can do this with an old pencil.

Solids which have the same shape throughout their length, like the cube and the pencil, are called **PRISMS**.

Answer these questions :

1 Is this cardboard box a prism ?

Give reasons for your answer.



2 Is a tetrahedron a prism ?

Give reasons for your answer.

Here are some more prisms.

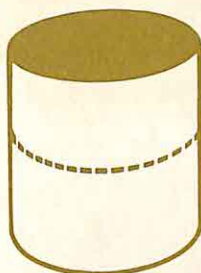
Imagine you were to cut each one in half along the dotted line, making your cut parallel with the end face.

3 Draw the shape of each cut face.

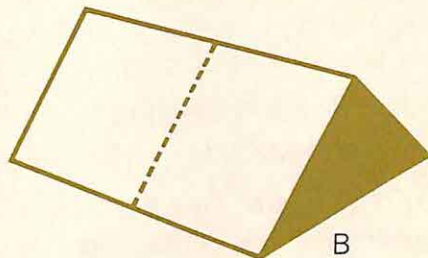
4 Is it the same shape as the end face ?

5 Where have you seen these prism shapes used ?

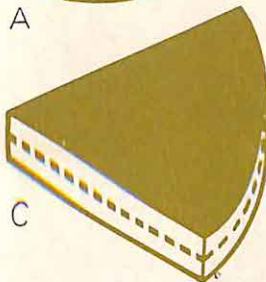
6 Find some more examples of prisms and draw them. Shape (A) is very common and is called a **CYLINDER**. How is it different from the other prisms ?



A



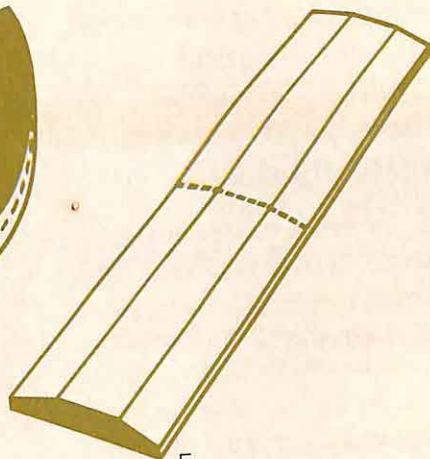
B



C



D



E

A cylinder is a prism with curved ends.

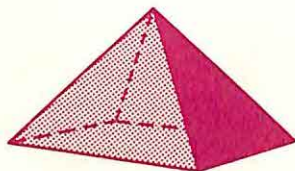
Which of these are cylinders ?



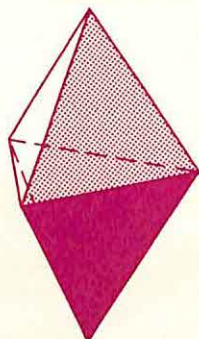
Some other solids

Here are some more solids you can make using equilateral triangles.

This is a pyramid with a square bottom.



This solid can be made quite easily by sticking together two tetrahedra.



How many triangular faces must it have?

Construct the net of this shape. Begin by drawing the square.

Make it and then answer these questions.

1 Is it a prism? Give a reason for your answer.

2 Add together the number of faces and vertices. Compare the number of edges. Do you find what you found before?

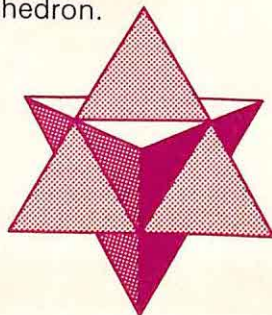
3 Can you make a net of this solid?

The net should only have six triangles.

It can be formed by sticking a tetrahedron to each face of an octahedron.

Colour each face as shown in the diagram and you will find you have a most attractive decoration.

This very attractive solid is called the stellated octahedron.



Volume

Make another 1 inch cube but leave the top face off.

Fill the cube level to the top with sand.

What is the volume of the sand in the cube?

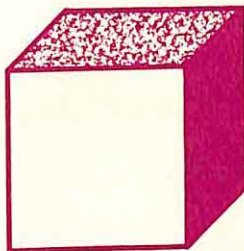
Find the volume of several small objects, such as small tins and boxes, school milk bottles, or egg cups.

Fill each one with sand and find how many inch cubes you can fill with each.

Make a table to show the volume of each container, like this.

Each edge of the cube is 1 inch long and each face is a 1 inch square.

If we wanted to describe the bulk or size of the cube we would speak of its VOLUME. The volume of the cube is 1 cubic inch.



Name of object	Volume

Supposing we had to find the volume of a solid object such as a large marble, a penknife, a rubber, or a stone. We could not fill the object with sand to find its volume.

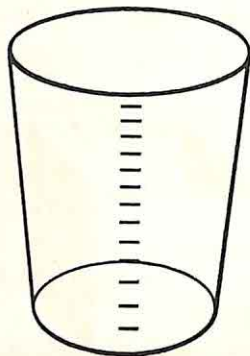
Here is a method we could use.

Pour a cubic inch of sand into a container such as a small jam jar or small glass. Shake the container so that the surface of the sand is level.

Mark the level of the sand on the outside of the container.

Add another cubic inch of sand and mark the level.

1 Continue adding cubic inches of sand and marking the levels. Why do the marks on the glass in the diagram get nearer together?



Pour enough water into the container to reach the first line you drew.

2 What is the volume of the water you have poured in?

Fill the container with water up to one of the lines which is about half-way up the side.

Drop in the solid whose volume you wish to find. Make sure all the solid is under water.



What happens to the water?

The rise in water shows the increase in volume caused by the object, and this rise can be read off on the side.

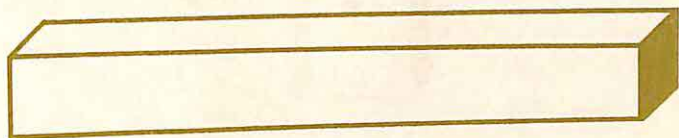
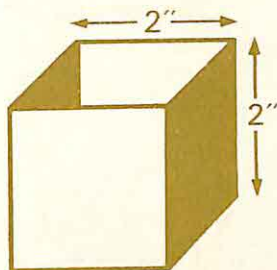
Find the volume of several objects in this way.

Record what you find in a table, as you did before.

We can find the volume of simple solids such as cubes and cuboids in another way without using sand or water.

Let us find the volume of this box.

Make the box from cardboard or graph paper.



1 Discover how many inch cubes it will hold. You may be able to combine with a friend to do this.

Estimate the answer first.

2 What is the volume of the box?

Now take the cubes out of the box and arrange them to make a line, like this.

3 What is the volume of the shape you have made? Is it the same as the box? Why?

Arrange the cubes in different ways and draw the shapes you make.

Is the volume of each shape always the same?

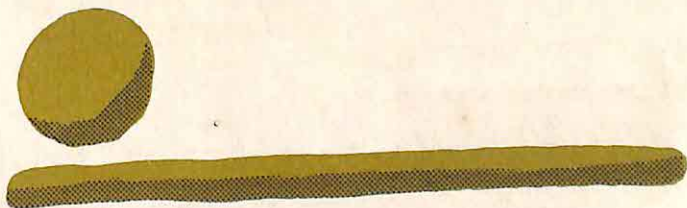
In *Pattern, Area and Perimeter*, another booklet in this series, we discovered that the same area can have many different shapes. We have now found that the same volume can also take different shapes. You can show this in another way.

Make an inch cube out of Plasticine or clay.

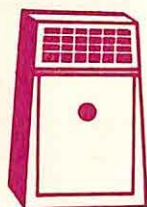
4 Roll it into a ball. What is its volume?

5 Now roll it into a long cylinder. Is its volume the same?

Make other irregular shapes with the same volume from your piece of Plasticine.



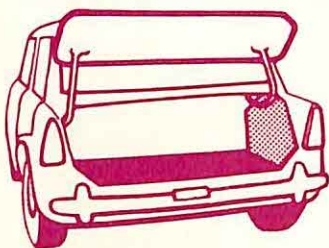
Make a cubic foot from cardboard. Find out how many cubic inches there are in a cubic foot.



Just as we measure large areas in square feet and square yards so we measure large volumes in cubic feet and cubic yards.

This stove has been made the right size to heat a room of 2000 cubic feet. If the room were very much larger, the stove would not generate enough heat to warm it in very cold weather.

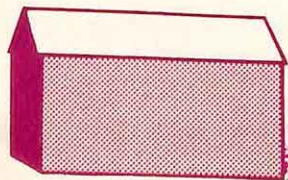
If your family has a car, try to find the volume of the luggage boot.



The boot of this car holds 12 cubic feet of luggage. We can see the number and size of the boxes it would hold.



Still larger volumes are measured in cubic yards. This lorry is delivering 6 cubic yards of ready mixed concrete to make a drive up to the garage.

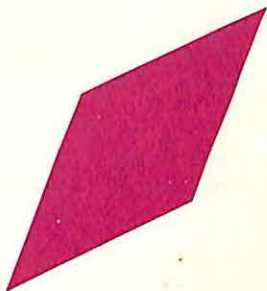


Find out some other things whose volume is measured in cubic feet and cubic yards.

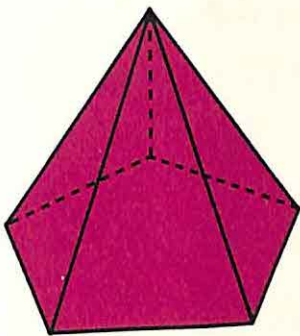


Puzzle Pages

- 1 Find four possible names for this shape.

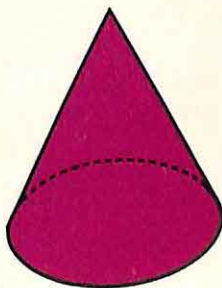
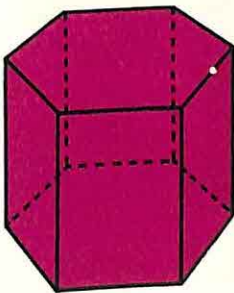


- 2 Make the net of this pyramid which has a pentagon at the bottom.

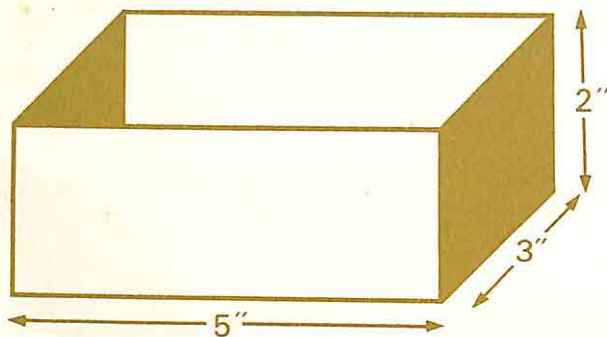


- 3 A solid has 12 faces and 20 vertices. How many edges does it have?

- 4 Which of these solids are prisms?



5 What is the volume of this box?



6 Is it possible to make a solid with 5 triangular faces?

Longmans, Green & Co Ltd
48 Grosvenor Street
London W1

Associated companies, branches and
representatives throughout the world

© Stuart E Bell 1967

First published 1967

The author has prepared these books with the help and advice of some of the members of the School Mathematics Project who have been introducing new ideas and new spirit into mathematics in the early years of the secondary school course. They feel that some of these ideas and approaches are relevant to children in the primary schools; such children would be thereby more readily equipped for the pattern of the secondary school course which is now emerging. In particular, the author gratefully acknowledges the close collaboration of Mr D J Holding of Exeter School, who is one of the principal S M P authors.

